5. (5 pts) A transmitter, operating at 1800 MHz, provides 15 W to an antenna having 12 dB gain. The receiver antenna has a gain of 3 dB and the noise floor for the receiver is -121 dBm. Measurements show that the environment can be modelled by a path loss exponent n= 4 with a standard deviation of σ =8 dB, where the reference distance d_0= 1 km. Please find the maximum Tx-Rx seperation that will ensure that a SNR of 20 dB is provided for 95% of the time.

Min rec. power = -121 dBm + SNR = -101 dBm

rec. power at d_0: $P_r(d_0) = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d_0^2}$

with $d_0 = 1$ km, $P_t = 15$ W, $\lambda = 0.167$ m, $G_t = 12$ dB $= 15.9$, and $G_r = 3$ dB $= 2$;

$P_r(d_0) = 8.42 \times 10^{-8}$ W $= -70.7$ dBW $= -40.7$ dBm

Now: $\overline{P_r(d_0)} = P_r(d_0) - 10n \log\left(\frac{d}{d_0}\right)$

$\therefore \log\left(\frac{d}{d_0}\right) = \frac{P_r(d_0) - \overline{P_r(d)}}{10n} = -1.017 - \frac{\overline{P_r(d)}}{10n}$ (dBm)

But $P(P_r(d) > -101 \text{ dBm}) = Q\left(\frac{-101 - \overline{P_r(d)}}{8}\right) = 0.95$

$\therefore$ let $\frac{-101 - \overline{P_r(d)}}{8} = \alpha \Rightarrow \overline{P_r(d)} = -101 - 8\alpha$ dBm

Then $\log\left(\frac{d}{d_0}\right) = -1.017 - \frac{-101 - 8\alpha}{40} = 1.508 + \alpha/5$

Now $0.95 = Q(z) = 1 - Q(-\alpha) \Rightarrow Q(-\alpha) = 0.05$

$\therefore -\alpha = 1.65$

$\therefore \alpha = -1.65$

$\therefore \log\frac{d}{d_0} = 1.508 + (-1.65)/5 = 1.178$

$\therefore d = 15.07$ km

Ans: